GPS gap repair

Datawell - Oceanographic Instruments

Method and program to repair GPS gaps in DWR-G wave buoy data (extended version)

In 2002 Datawell introduced a new technique to measure 3-dimensional motion with a single GPS receiver. The DWR-G wave buoys and sensor package, and the MOSE-G1000 motion sensor are all based on this measurement technique. It features centimetre precision up to periods of 100 s at an attractive price-performance ratio. Occasionally gaps may occur. High GPS antenna accelerations, tilting away from low elevation GPS satellites and waves washing over the GPS antenna and blocking the GPS signals, all can lead to missing wave measurements, gaps. This document describes a method to largely repair these gaps, thus extending the use of DWR-G buoys. For your convenience the method has already been implemented in an associated program (GPSrepair.exe) that will directly repair HXV- or RDT-files.

1. Idea

Figure 1(a) shows a timeseries of the heave or vertical excursion and the status from an exemplary HXV-file. Two gaps are present as marked by the status 128. Initially, GPS gaps are flagged in the least-significant-bit of the north, but here they have been converted to a status offset of 128 by a conversion program available from Datawell Sales (GPSconv.exe). Occasionally status 4 (sync error) appears. At the first gap not much happens, but at the second gap the vertical is clearly distorted. The distortion bears the signature of the GPS digital filter in the buoy, compare with the blue line in Figure 2(a). This brings up the idea to try to recognize the filter pattern in order to identify gaps and quantify the missing measurements in the gaps.

Mathematically we correlate the vertical with the known GPS filter. The correlation result is then supposed to display a marked peak at the gaps, see Figure 1(b). With the proper normalization of the correlation, the peak height indicates the value of the missing measurement in the gap. Note that indeed the peak at the first gap is much smaller. Apparently, the missing measurement is much smaller overthere and so is the amount of filter signature. After repair the filter signature has largely been eliminated, Figure 1(b).

2. Mathematics

We will now rephrase the above idea in mathematical terms and after that in mathematical formulas.

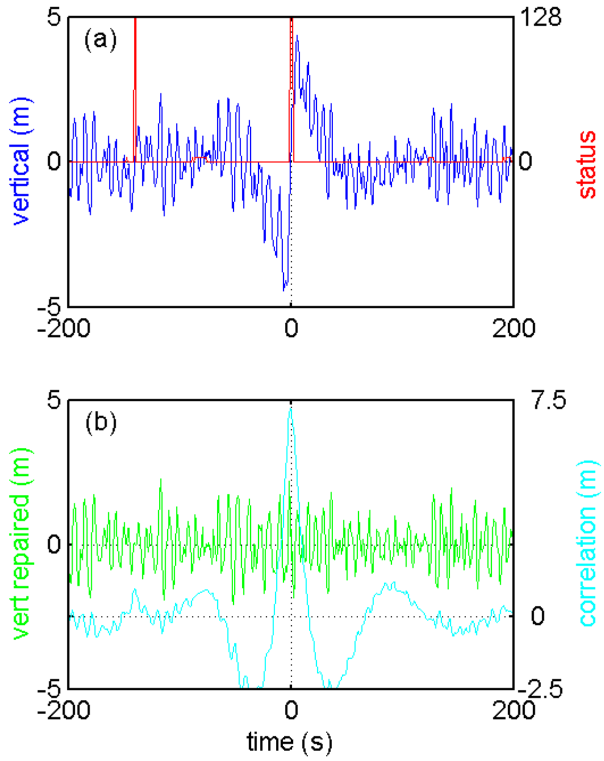

Figure 1. (a) Timeseries of vertical excursions with two flagged gaps indicated by status 128. (b) Correlation result with a small and large peak at the first and second gap and repaired vertical.

Under normal circumstances, i.e. no gaps, GPS measurements are input at a given sample rate into the buoy's GPS digital filter which, after a certain filter delay, ouputs the vertical, north and west wave excursions. In case of a missing GPS measurement a zero is entered in the filter array. One could also say that the right measurement is entered, however, with a term of equal size but opposite sign added. In the latter picture the right wave result will emerge, however, with the response of the filter to the extra term added. The response of the filter to the extra term is in fact the impulse response of the filter, which is the filter itself. Hence, it is no coincidence that we found the signature of the GPS digital filter in Figure 1, it is exactly what one would expect. One way to recognize this GPS filter pattern is to use correlation, for which we require a suitable set of a correlation coefficients. Ideally, the correlation result should display a peak at the location of the gap with a peak value equal to the missing measurement. Furthermore, it should fall of rapidly away from the peak and remain small for waves without gaps. One last remark about gaps. The filter input inside the buoy actually is zero during gaps, but due to the filtering process the filter ouput (the HXV- and RDT-file content) is not, as Figure 1(a) also demonstrates.

In the mathematical formulas the starting point is the sampled vertical, north or west signal s_i in the HXV- or RDT-file. It consists of a sampled wave signal sw_i and possibly some response signals due to gaps sg_i

$$s_i = sg_i + sw_i \qquad (1)$$

Correlating with some set of correlation coefficients c_i (length $2N_c+1$, $-N_c \le i \le N_c$) produces the correlation result r_i

$$\begin{aligned} r_i &= \Sigma_j\, s_{i+j}\, c_j & (2a)\\ &= \Sigma_j\, sg_{i+j}\, c_j + \Sigma_j\, sw_{i+j}\, c_j & (2b)\\ &= rg_i + rn_i & (2c) \end{aligned}$$

Of the two contributions rg_i will be used to repair possible gaps, while rn_i, the correlation result with the waves, acts as noise that will limit the precision with which the gaps can be repaired. In the absence of waves (no noise, rn_i=0) any series of gaps can be repaired exactly to within numerical precision. Alternatively, in the absence of gaps (rg_i=0, gaps absent, avoided or repaired) rn_i indicates the gap repair precision.

Applying the convolution theorem — the Fourier transform of the convolution of two functions equals the product of their individual Fourier-transforms — to Equation (2b-c) in the absence of gaps (sg_i = 0), results

$$RN_k = SW_k\, C_k \qquad (7)$$

Here upper case indicates the Fourier transform of the lower case variables. Note that the convolution theorem has actually been applied to a correlation fomula, Equation (2b). In this case this is allowed because of the skew-symmetry of the correlation coefficients c_i, although this does introduce an extra minus-sign. With Parseval's theorem — the sum of the squared modulus of a function is equal to the sum of the squared modulus of its Fourier spectrum — the expression for the variance of the noise becomes

$$\begin{aligned} \sigma^2_{rn} &= 1/N\, \Sigma_i\, (rn_i)^2 & (8a)\\ &= 1/N^2\, \Sigma_k\, |RN_k|^2 & (8b)\\ &= 1/N^2\, \Sigma_k\, |SW_k|^2\, |C_k|^2 & (8c) \end{aligned}$$

with N the number of rn_i samples or RN_k Fourier coefficients. The aforementioned minus-sign has now disappeared in the quadratic. Equation (8c) has the following implication. Merely a visual inspection of the power spectral density (PSD) of a particular set of correlation coefficients in relation with the wave PSD without gaps (absent, avoided or repaired) suffices to estimate the gap repair success. Minimum gap repair noise σ_{rn} will result if $|SW_k|^2$ is small when $|C_k|^2$ is large and vice versa.

3. Implementation and improvements

In order to recognize the GPS digital filter g_i (length $2N_g+1$, $-N_g \le i \le N_g$) an infinite number of sets of correlation coefficients can be chosen. Each with its own correlation function

$$f_i = \Sigma_j\, g_j\, c_{i+j} \qquad (|i+j|>N_c,\ c = 0) \qquad (9)$$

Normalization requires $f_0 = \Sigma_j\, g_j\, c_j = 1$ for zero shift i=0, so that a gap of 1 m produces a correlation result of 1 m. Different criteria, e.g. delta-peak correlation function or minimum correlation with a given wave signal, produce different sets of correlation coefficients. Of the infinite number of correlation methods we will select only two that follow from common sense.

As a first choice for the correlation coefficients we take the GPS digital filter itself and normalize the coefficients

$$c_i = g_i / \Sigma_j\, g_j\, g_j \qquad (10)$$

We will call this the direct method and the correlation coefficients (magnified by 10), their power spectral density and the correlation function are shown in Figure 2 (in blue). Note that the correlation function displays a rather broad peak — the first zero-crossing lies at 16 seconds from the peak — which will make it difficult to discern two gaps close to each other. Otherwise put, the direct method is susceptible to cross-correlation. By the following reasoning we can reduce the cross-correlation. Note that c and g are identical to within a normalisation factor so their correlation is in fact an

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autocorrelation. By definition the autocorrelation function is symmetric. This leads to a relatively sharp slope discontinuity in an otherwise smooth function. To sharpen the correlation function one can therefore take the difference of the slope before and after each point.

$$d_i = (c_{i+1} - c_i) - (c_i - c_{i-1}) \qquad (11)$$

After proper normalization such that $\Sigma_j\, d_j\, g_j = 1$ we obtain an alternative set. Figure 2 also shows the coefficients, their PSD and the correlation function of this "differential" method (in red). Indeed the correlation function peak is much sharper now. As a result two nearby gaps can now be discerned. Although from the point of cross-correlation the differential method is preferred, it can not be said beforehand which method yields the smallest gap repair noise. This also depends on the wave spectrum. For a wave spectrum looking like the coefficient spectrum of the differential method ($|D|^2$), direct correlation (with c) would yield better results, and vice versa. Recall that $|SW_k|^2$ should be small when $|C_k|^2$ is large and the other way around.

In practice, noise will compromise the undisturbed curves in Figure 2. Instead of simply accepting the noise we can modify the correlation coefficients to improve the result in the presence of noise. This technique is called optimal Wiener filtering and it works a bit like Equation (8c). A common way to design a filter is to define the desired transfer function in the frequency domain (X) and inverse Fourier-transform to the time-domain (x). The convolution $x*y$ now filters the time-domain signal y with filter x. In the presence of noise Wiener suggests to weigh the original X with the noise and signal power spectral densities $|N|^2$ and $|Y|^2$ as follows

$$X' = X\,|Y|^2/(|Y|^2+|N|^2) \qquad (12)$$

The weighing is such that frequencies where the noise dominates the signal contribute less than frequencies where the signal dominates the noise. In our situation Wiener filtering implies

$$C' = C\,|SG|^2/(|SG|^2+|SW|^2) \qquad (13)$$

and similar for the differential method (D'). The inverse Fourier transform will produce the optimal correlation coefficients (c'_i and d'_i). Problems that arise are that SG

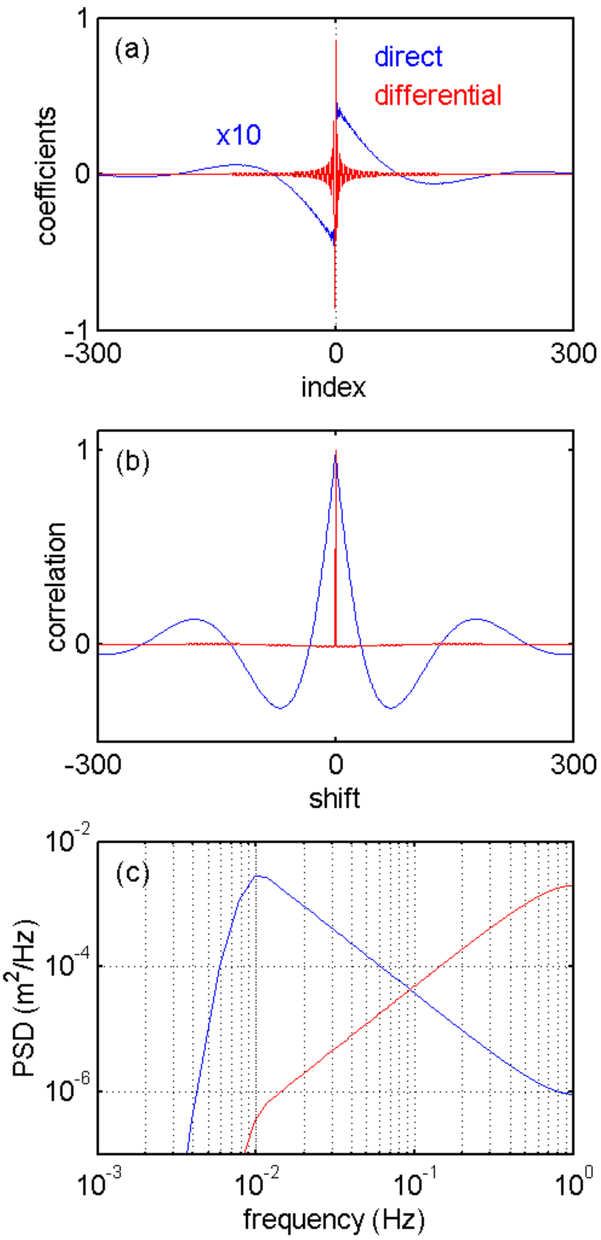

Figure 2. Correlation coefficients (a), correlation function (b), and the power spectral density of the correlation coefficients (c), for the direct and differential method. The tails in (a) and (b) are not shown completely.

is not known beforehand and SW initially will also contain gaps. We will assume a single gap of 1 meter and solve for the gaps with the resulting optimal filter C'. In the next iteration we can use the new reasonably gapfree SW to improve the solution for the gaps by recalculating the optimal filter with the new *SW*. And so on.

4. Evaluation

Now that we have developed a method to repair gaps, it is time to see how good it works. First we are faced with the paradox of requiring a GPS buoy dataset including some gaps with knowledge about the unknown measurements in the gaps. One way out is to use a dataset without gaps and introduce known gaps. A file with the GPS filter coefficients at 1.28 Hz is included in the repair program files for you to pursue this path and evaluate the program yourselves. Just add the array of coefficients one to one to an equally long stretch of data in your own HXV-files, add 128 to the status at the centre

$$hxv_{i+j} = hxv_{i+j} + g_j \qquad (-N_g \le j \le N_g) \qquad (14a)$$

$$status_i = status_i + 128 \qquad (14b)$$

and see if the program correctly determines the size of the gap. You can also multiply by a constant or even add multiple gaps. Another way out is to use, as we will, a dataset of a combined GPS and MkII directional Waverider. The natural gaps in the GPS dataset can be repaired, while the result can be evaluated by comparing with the MkII dataset. Such an experiment was done on the North Sea in January 2002. Typically gaps occurred during peak tidal currents (1 m/s) and high waves (H_σ=4 m). For evaluation a 10 hour dataset under the given conditions has been selected. The sample rate was 2 Hz.

Merely showing histograms of unrepaired and repaired gap errors is not very informative and is delayed until later. More insight can be gained by calculating the gap repair precision in advance. Instead of using Equation (8c) to calculate the gap repair noise σ_{rn} directly, we will use Equation (2c) to calculate r_i first and from it the standard deviation σ_r. By iteratively repairing the gaps the signal due to gaps (rg_i) will approach zero to within

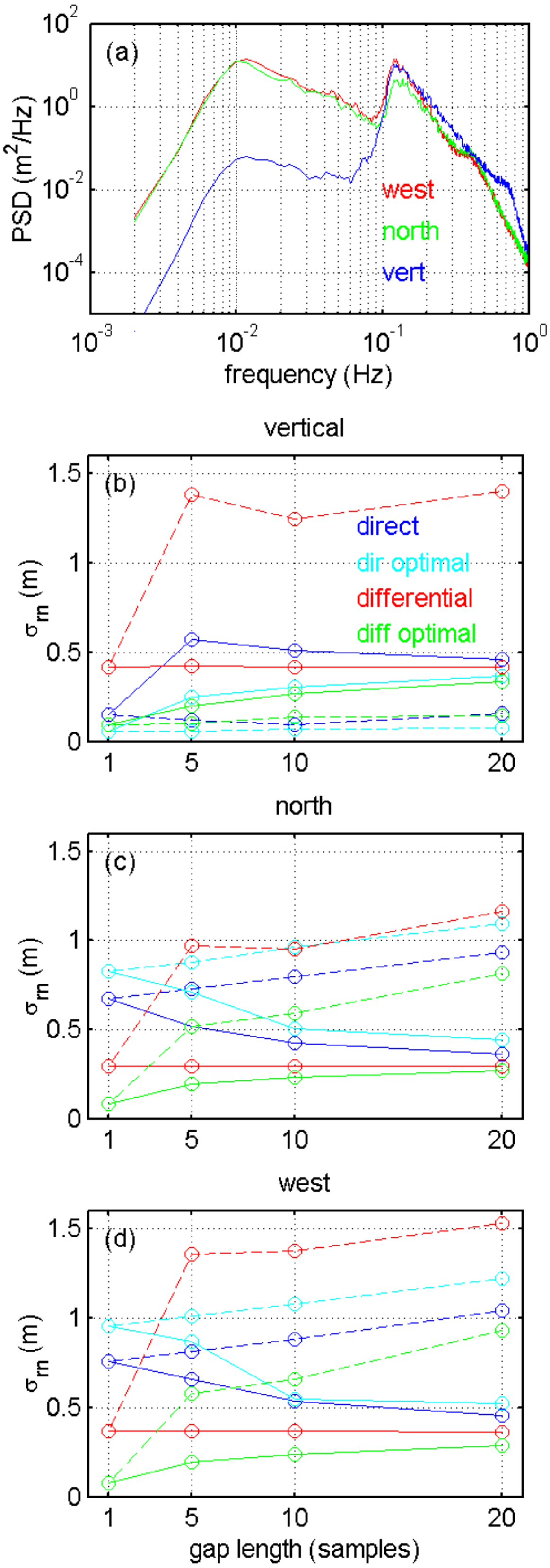

Figure 3. Gap repair precision against gap group length for four correlation methods and three directions (b-d). Both the net (dashed line) and individual (solid line) gap repair precision are given. The average wave power spectral density after repair is shown in (a) for a qualitative estimate of the repair precision of the various methods.

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the order of σ_{rn} and rn_i and σ_{rn} are found for the whole 10 hour dataset. There is however one more issue, a group of gaps may have varying length and the gap repair precision will vary accordingly. For gaps longer than one sample the cross-correlation, even for the differential method, must be eliminated first. The cross-correlation between individual gaps at locations i and j inside a single group of gaps is determined by their distance i−j and the correlation function f

$$M_{ij} = f_{i-j} \tag{15}$$

For a gap group of l samples length, matrix M is of dimension lxl. To eliminate the cross-correlation from the original l correlation results (r_i), the formula below should be applied

$$r'_j = M_{ij}^{-1} r_i \tag{16}$$

r_j' contains the l gap repair values for the l individual gaps in the group, after having eliminated the cross-correlation. In the absence of noise (no waves, $rn_i = 0$) the uncorrelated gaps would be recovered exactly from the correlation result with the gap signal rg_i, even for a broad correlation function peak. By considering only the noise again (no gaps, $rg_i = 0$), we find the gap repair precision for gaps of a given length. Apart from the repair precision for individual gaps also the repair precision of the net gap $\Sigma_j r_j'$ (j=1..l) is determined.

For groups of gaps of various lengths, the precision of the individual gap repairs and the precision of the net gap repair of the whole group are shown in Figure 3(b-d). Here gap group lengths of 1, 5, 10 or 20 (the maximum that the program repairs for 2 Hz data) samples are investigated. Given the length l the whole 10 hour dataset has been analysed in consecutive sets of this length. As the wave spectra differ for vertical, north and west directions, see Figure 3(a), so do the gap repair precisions and each direction must be evaluated separately. In general, one can conclude that Wiener optimal correlation is usually better than normal correlation. An exception is formed by the direct optimal method for north and west directions, where Wiener optimal filtering fails. Apparently, the limited gap repair precision for small gap group lengths prevents iteration to a better solution than the direct normal method. As the program also outputs the gap repair precision for gap groups of various lengths, the user can simply check if this is the case. Furthermore, for the vertical direction the differential method is better at estimating individual gap repairs, while the direct method excels in estimating the net gap repair. This finding is consistent with the width of the correlation function of both methods. A broad peak averages the noise and produces an improved net gap repair estimation. On the other hand its individual gap repair estimate will be highly sensitive to noise from neighbouring individual gaps (check out M^{-1} for f_0=1, f_1=0.9).

So far the repair methods have been evaluated relative to each other. However, the question remains whether even the best of the presented repair methods offers an improvement over not repairing at all. It appears that the normal differential method can serve as an absolute reference for repair evaluation. The delta-peak correlation function of this method implies that from the filter output (north, west, vertical excursions) the original filter input can be recovered nearly exactly. Hence, the gap repair precision σ_{rn} equals the variation of the input signal. As gap repair tries to predict the missing input, it is clear that an alternative repair method only constitutes an improvement if it outperforms the normal differential method in terms of σ_{rn}. Otherwise repair might as well be left. One can also reason as follows. During gaps actually zeros are input into the buoy's digital filter g_i. As said the normal differential method recovers these input zeros and uses them for repair. Of course repairing all gaps with zeros is the same as not repairing at all. Figure 3(b-d) learns that improvement depends on coordinate direction, gap length and net or individual gap repair.

The quantitative results have been presented in Figure 3(b-d). However, from the wave power spectral densities in Figure 3(a) and the PSD of the direct and differential correlation method in Figure 2(c), we could already have drawn some qualitative conclusions. Also refer to the discussion of Equation (8c). The high PSD at low frequencies for north and west as compared to vertical already foretells that the gap repair precision of north and west will be worse. On the other hand the repair precision of the differential method will not differ too much as the high frequency side of the wave spectra is comparable for all three directions. In this qualitative manner it is not so easy to predict which method, direct or differential, will perform better.

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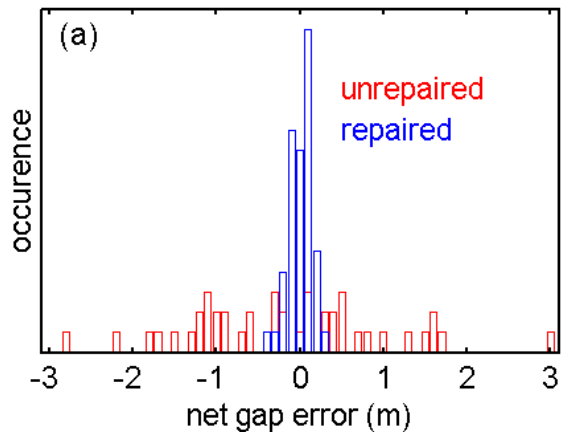
As a first evaluation step we predicted the gap repair precision of four different methods in three coordinate directions. The next step is to check if the effective repairs in our experiment support the theory. Figure 4 shows the net gap error (a) and the individual gap error (b) for a number of groups of gaps before and after repair. This is possible by referencing the repaired and unrepaired GPS dataset to the MkII dataset. Only the vertical direction result is plotted since it offers the highest precision of the GPS and MkII datasets. Furthermore, only the Wiener optimal differential method, one of the most successful methods, is considered. Net gap errors up to ±3 m have been repaired nicely. After repair the net gap error can be characterised by a normal distribution with a standard deviation of roughly σ=0.1 m. This agrees very well with the prediction based on the GPS dataset alone, Figure 3(b) dashed green line, without help from the MkII set. Typically, the maximum net gap errors that remain amount to ±4σ.

Even individual gap errors are accurately repaired. However, the histogram after repair displays long tails extending up to ±0.9 m, whereas the net gap error histogram after repair is confined to ±0.3 m. Actually the vast majority of gap groups falls in two length categories: gap groups of 1-2 samples and gap groups of 7-8-9 samples length. By selecting only the small length gap groups the tails disappear. In fact the histogram can be characterised by two superimposed normal distributions with standard deviations of roughly σ=0.1 m and σ=0.2 m and both with extremes of ±4σ. Indeed the solid green line in Figure 3(b) indicates that the gap repair precision of individual gaps increases with gap group length, attaining the observed values for the respective gap group lengths. On the contrary, the dashed green line for the precision of net gaps remains at a constant level for all gap group lengths. The important conclusion is that all the above findings are along the lines of our theory.

5. Effect on wave parameters

Especially wave parameters in the time domain, such as significant wave height (H_σ), maximum wave height (H_{max}), zero-up crossing period (T_{zu}), etc will benefit from repair. For best results one should minimalize the net gap group error. Individual gap errors are less relevant, provided that individual waves, defined as the heave between two zero-up crossings, are skipped for



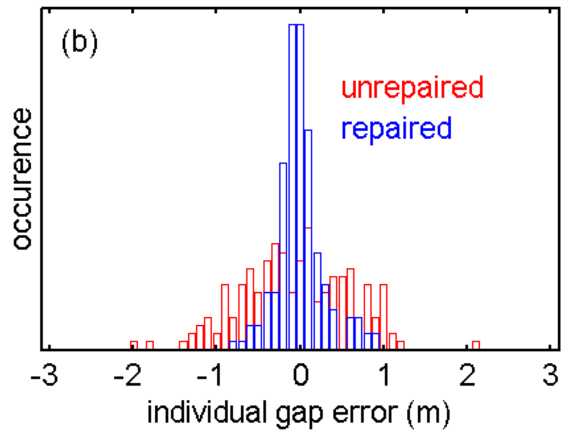


Figure 4. Repair results for the given dataset of 10 hours, which contains 140 individual gaps in 49 groups. The histograms show the vertical net gap error (a) and the vertical individual gap error (b) before and after repair.

analysis if they contain gaps. The degree of improvement depends on the number of the gaps and the repair precision relative to the wave heights. For the dataset under consideration we have H_σ=4 m, i.e. σ_{heave}=1 m. Comparing with σ_{repair}=0.1 m learns that the remaining gap repair error is an order of magnitude smaller than the typical heave. Also note that σ_{repair} roughly scales with σ_{heave} and H_σ, see Equations (2b) and (8c), so that the conclusions apply more in general. To give an idea, the remaining gap repair error of a single, repaired gap group in a half hour dataset will produce a significant wave height deviating at most 0.1 % from the gapfree data value. Outside repaired gap goups slowly oscillating tails of the buoy's digital filter g_i with amplitude of order σ_{rn} remain. This will introduce small offsets for individual waves. Values and precision of individual wave periods and heights will only be affected marginally. As a result, T_{zu} and H_{max} are still very accurate, although H_{max} may be the height of the second or third highest wave.

Spectral wave parameters will also improve, but it should be noted that these do not suffer so much from gaps. Gaps produce a sloping background in the wave power spectral density increasing for lower frequencies. For example, a remaining gap repair error of 0.1 m produces a background like the blue line in Figure (2c). Even for an unrepaired gap of 1 m, with a sloping background a 100 times as large, the wave PSD usually dominates the gap PSD contribution at wave frequencies (10-20 s). This applies to vertical as well as north and west components. Therefore spectral parameters at the peak of the spectrum, such as peak power spectral density, peak wave direction, peak spread and peak wave ellipticity, are not changed much by the presence of gaps, repaired or not. However, gap repair will improve the low frequency part of the spectrum.

6. 1.28 Hz and 2 Hz sampling frequency

The foregoing evaluation focussed on data sampled at 2 Hz, whereas HXV- and RDT-files contain data sampled at 1.28 Hz. Internally the DWR-G buoys sample at 2 Hz and this signal is converted into 1.28 Hz to conform to the long-existing Datawell data format. The sample rate conversion is done by means of decimation filters in the buoy. However, the decimation step contains an ambiguity and may lead to several slightly different effective GPS digital filters. In practice, one does not know which filter the buoy is actually using. The GPS repair methods presented here rely on a priori knowledge of the GPS filter. As for the direct and direct optimal methods a slightly differing filter is acceptable. The differential method on the other hand, being the second-derivative of the direct method, is highly susceptible to even the slightest differences in coefficients as one can imagine. Therefore, the differential and differential optimal methods are not possible for 1.28 Hz data and the program does not accept these combinations.

7. GPSrepair.exe program

As mentioned Datawell offers a ready-made program to repair gaps as described here, called GPSrepair.exe. The program can handle HXV- and RDT-files. As the gap response extends over the GPS filter length, the datafile following the present datafile is required to repair the end of the present datafile. For this reason it is preferred to feed the program with batches of consecutive files. Furthermore, RDT-files are preferred over HXV-files, since the latter may contain missing vectors due to transmission problems. Still HXV-files with a limited amount of missing vectors can be handled by the program. The repaired data are saved per half hour file in HXV- or RAW-format, respecting the original start and end, and with a $ prefix to the original filename. RAW output files are used for RDT input files because the RDT format does not offer a status field. Apart from the repaired files also three logfiles are saved, one for each coordinate direction. The logfiles list the gap location (line index) and size of all repaired gaps and the net and individual gap repair precisions for four different gap group lengths per input file. Also various warnings are logged, e.g. if a group of gaps is too long to repair or if too much missing vectors are encountered. Maximum gap group length is 20 samples at 2 Hz or 13 samples at 1.28 Hz. Besides 1.28 Hz data the program also accepts 2 Hz data. In the future DWR-G buoys may offer a configurable output data rate (1.28 or 2 Hz). Four repair methods are offered: direct, direct optimal, differential and differential optimal. The latter two are not available for 1.28 Hz. It is suggested to use optimal methods if possible. For each coordinate direction a different method can be selected. Finally, the program contains a helpfile with detailed information.

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