Platform stability

Datawell - Oceanographic Instruments

Tilting and swinging of the Datawell stabilized platform

The essence of the Datawell wave measuring buoys is its stabilized platform. Almost insensitive to rotations and translations the platform remains horizontal. Once the stabilized vertical reference is obtained, the measurement of wave height through vertical accelerations is straightforward.

The platform may be represented by a swing of 400 m length, having a natural swinging period of 40 s. Just like this swing, a temporary horizontal acceleration may slightly tilt the platform and set it swinging with its natural period. Due to buoy movement in the waves or in a calibration rig a different forced swinging period may result. This document quantifies the measurement errors introduced by such a swinging platform and it will demonstrate that the wave height and direction are still correctly measured by the Datawell (Directional) Waverider buoys.

Theory

Suppose the (Directional) Waverider platform is tilted at an angle φ with the vertical. As a result the measured 'vertical' acceleration A_z referenced to the platform now deviates from the true vertical acceleration A_v, see Figure 1(a).

$$A_z = A_v \cos(\varphi) \qquad (1)$$

Directional Waveriders also measure 'horizontal' accelerations with respect to the buoy frame, the x,y-plane defined by the fender. When the buoy (not the platform) is tilted, these measured accelerations A_x and A_y will contain a gravity component that is eliminated with the known pitch (π) or roll (ρ) angles, yielding the true x,y-accelerations A_x' and A_y', e.g. assuming $\rho = 0$ and $A_y' = 0$

$$A_x' = A_x - g \sin(\pi) \qquad (2)$$

g is the gravitation acceleration. After projecting the buoy x-axis onto the horizontal direction, the horizontal acceleration is obtained

$$A_h = A_x' \cos(\pi) \qquad (3)$$

see Figure 1(b). Note that the buoy can not discriminate between a tilted hull and a tilted platform. In both cases a pitch (or roll) will be measured and the same rules will be applied. In the former true tilt situation a compensating x-acceleration is measured $A_x = g \sin(\pi)$, correctly giving $A_x' = A_h = 0$. In the latter false tilt situation no x-acceleration is measured $A_x = 0$, hence erroneously $A_x' \neq 0$ and $A_h \neq 0$.

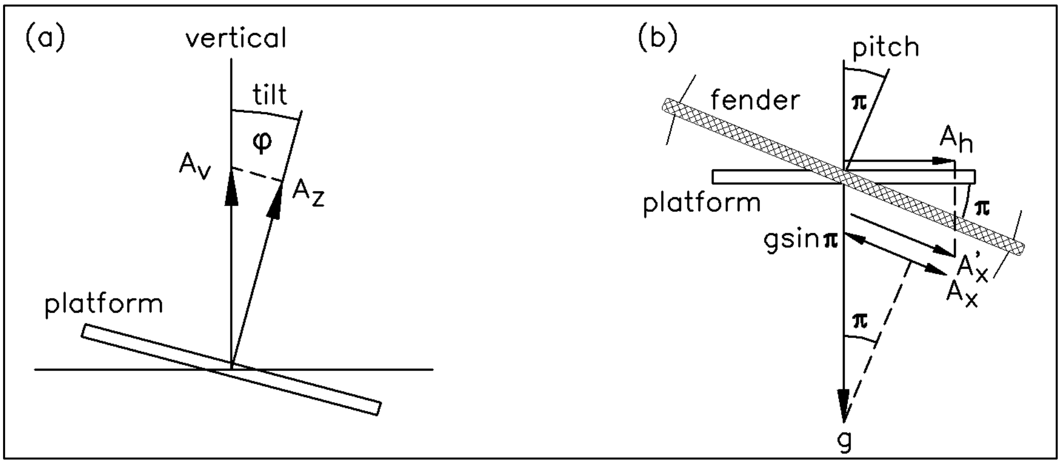

Figure 1. Illustration of (a) incorrectly measured vertical accelerations due to a faulty platform angle and (b) measured and true x-accelerations due to a pitch.

Examples

In the examples below the following approximations and equality are used

$$\sin(\varphi) \approx \varphi$$
$$\cos(\varphi) \approx 1 - \tfrac{1}{2}\,\varphi^{2}$$
$$\sin^{2}(\varphi) = \tfrac{1}{2} - \tfrac{1}{2}\cos(2\,\varphi)$$

with φ in radians. Furthermore, due to filtering all constant terms are completely suppressed and therefore omitted in the examples below.

Example I: Tilt and vertical acceleration.
Given a static fault angle of 1°, the vertical acceleration error and consequently the heave error become: $1-\cos(1^\circ) = 1.5\ 10^{-4}$ or 0.015 %, which is negligible.

Example II: Swinging and vertical acceleration.
Let us assume a 40 s period swing with 0.1° angular amplitude, $\varphi = 0.1^\circ \sin(2\pi t/(40\ \mathrm{s}))$. Substituting this into (1) with $A_v = g$, applying the above equality and cosine-approximation and skipping the constant part, we obtain a false vertical acceleration

$$A_v \approx\ \tfrac{1}{4}\, g\ (0.1^\circ\ \pi/180^\circ)2\ \cos(2\pi t/(20\ \mathrm{s}))$$

Note the doubled frequency, halved period! This double integrates to a negligible $\tfrac{1}{4}\, g\ (0.1^\circ\ \pi/180^\circ)2\ ((20\ \mathrm{s})/(2\pi))^{2} < 0.1$ mm amplitude of heave modulation.

Example III: Swinging and horizontal acceleration.
A 40 s period and a 0.1° amplitude fault angle are again assumed. The erroneously calculated horizontal acceleration is

$$A_h \approx -\ g\ (0.1^\circ\ \pi/180^\circ)\ \sin(2\pi t/(40\ \mathrm{s})),$$

which, after double integration, yields a false horizontal modulation amplitude of $((40\ \mathrm{s})/(2\pi))2\ g\ (0.1^\circ\ \pi/180^\circ) \approx 71$ cm! Buoy filters will reduce this amplitude, as shown in Figure 2.

Final remarks

The above examples make it clear that measured wave height is unaffected by the swinging of the platform. Also the wave direction remains correct as the plane of movement of a circle and an ellipse are identical.

Instead of physically ‘searching’ for the direction of gravity, wave buoys using three linear and angular accelerometers mathematically search for the direction of gravity. If a second order model is used, they too will display these modulation effects. However, the final heave will contain the errors of all accelerometers, unlike the Datawell principle where the heave only contains the error of one single (vertical) accelerometer.

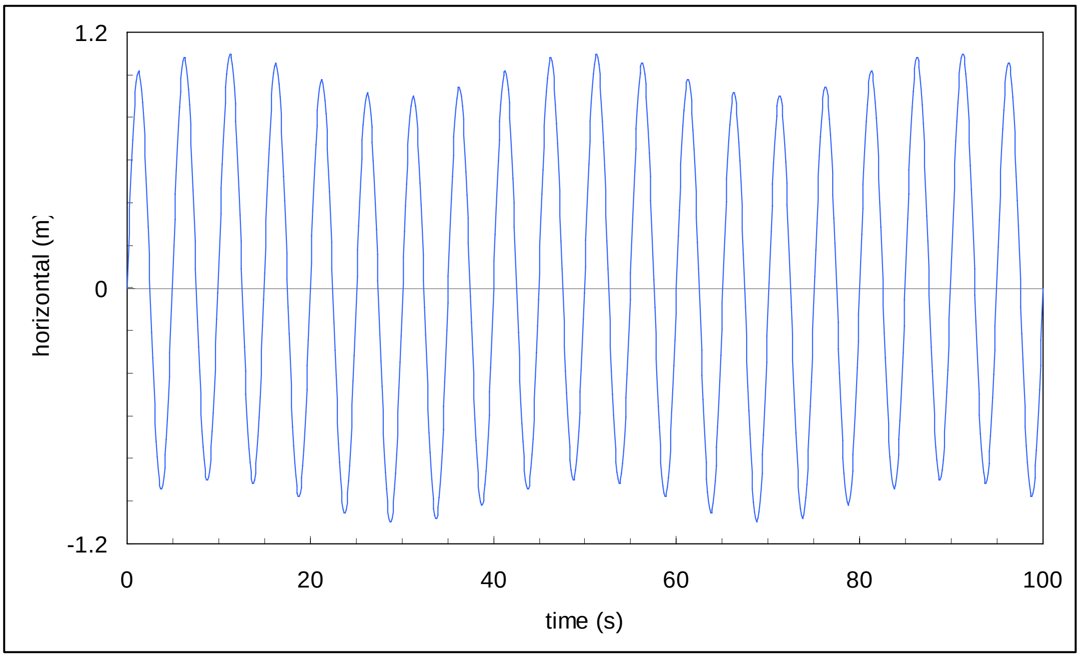

Figure 2. Horizontal component of a forced calibration rig motion of 1 m-amplitude 10 s-period with an additional 0.1 m-amplitude 40 s-period modulation due to a swinging platform.